Government autonomous college, Rourkela

Question Bank

Paper 403

Each question contains 1 mark/ 1.5 mark

1. Define Euclidian norm.
2. Define Uniform norm.
3. What is L^p norm.
4. What is L^2 norm.
5. Define least square approximation.
6. Define orthogonality.
7. Write Gram-Schmidt Orthogonalizing process.
8. Define pivoting.
9. When there is no pivoting is necessary.
10. Define pivoting, types of pivoting & write briefly about pivoting.
11. Define rate of convergence. Define asymptotic error constant.
12. Define weierstrass approximation.
13. Define Existence and uniqueness theorem.
14. Using Gram Schmidt orthogonalization process find $\emptyset_1$(x) on [-1,1] w.r.t weight function w(x)=1.

    Each question contains 2 mark.

15. Define error of approximation.
16. Write non uniform nodal points, polymorphism & error.
17. Write error in Linear Interpolation.
18. Write error in Quadratic interpolation for non-uniform nodal interpolation.
19. Write error for uniform linear interpolation nodal point.
20. Write error for Quadratic interpolation.
21. Write the optimum choice of step length error.
22. Write optimum value when |R.E.| = |T.E.| & Write optimal value when |R.E.| + |T.E.| = minimum.
23. Write the Richardson Extrapolation method.
24. Write the formula for $(\frac{\partial^2 f}{\partial x \partial y})_{(x_0,y_0)}$ also its order.
25. I = $\int_{-\infty}^{\infty} \frac{e^{-x^2}}{x^2+x+1}$ dx. find the value by using Gauss Hermite two point formula.
26. Write condition for Lipschitz Function.
27. Write formula for Gauss Hermite 2- point formula.
28. Write formula for Gauss Hermite 3-point formula.
29. Write the condition for stability of solution of system.
30. When the method is called absolutely stability.

31. When the method is called relatively stability.
32. When the method is called Unconditionally stability.

Each question contains 6 marks

33. Solve the equation

$$10x_1 - x_2 + 2x_3 = 4$$
$$x_1 + 10x_2 - x_3 = 3$$
$$2x_1 + 3x_2 + 20x_3 = 7$$

Using Gauss elimination method.

34. Solve the equation

$$x_1 + x_2 + x_3 = 6$$
$$3x_1 + 3x_2 - 4x_3 = 20$$
$$2x_1 + x_2 + 3x_3 = 13$$

Using Gauss elimination method.

35. Solve the system of equation

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

using the Gauss elimination method with partial pivoting.

36. Find the inverse of the coefficient matrix of the system

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

by the Gauss-Jordan method with partial pivoting and hence solve the system.

37. Consider the equation

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

Use the decomposition method to solve the system.

38. $f_1(x)=1$ & $W(x)= 1$ find L_1 norm.
39. $f_1(x)=1+ \sin(8\pi x)/2$ & $W(x) =1$ find L_2 norm.
40. $f = 1+ \alpha e^{-\alpha^2 x}$ $(\alpha = 100)$ on [0,1] find L_∞.
41. Obtain a linear polynomial approximation to the function $f(x) = x^3$ on the interval [0, 1] using the least square approximation with $W(x) = 1$.
42. Obtain the least square polynomial approximation of degree one and two for $f(x) = x^{1/2}$ on [0, 1].
43. Derive the least squares straight lines and quadratic fits for the discrete data (x_i, f_i), $i = 0,1,2,\dots,N$.

44. Obtain the least squares straight line fit to the following data

x	0.2	0.4	0.6	0.8	1
$f(x)$	0.447	0.632	0.775	0.894	1

45. Find the least squares approximation of second degree for the discrete data

x	-2	-1	0	1	2
$f(x)$	15	1	1	3	19

46. Use the method of least squares to fit the curve $f(x) = c_0 + (c_1/\sqrt{x})$ for the following data

x	0.2	0.3	0.5	1	2
$f(x)$	16	14	11	6	3

47. We are given the following values of a function of the variable t:

x	0.1	0.2	0.3	0.4
t	0.76	0.58	0.44	0.35

Obtain a least squares fit of the form

$$f = ae^{-3t} + be^{-2t}$$

48. Using the Gram-Schmidt orthogonalizing process, compute the first stage three orthogonal polynomials $P_0(x), P_1(x), P_2(x)$ which are orthogonal on [0, 1] with respect to the weight function $W(x) = 1$. Using these polynomials obtain a least square approximation of second degree for $f(x) = x^{1/2}$ on [0, 1].
49. Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ using i) trapezoidal rule ii) Simpson's rule. Obtain a bound of the errors.
50. Find the approximate value of $I = \int_0^1 \frac{\sin x}{x} dx$ using i) mid-point rule ii) two point open type rule.
51. Find the reminder of Simpson's three eighth rule

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)]$$

for equally spaced points $x_i = x_0 + hi, i = 1,2,3$. Use this rule to approximate the value for the integral

$$I = \int_0^1 \frac{dx}{1+x}$$

Also find the bounded error.

52. Determine a, b and c such that the formula

$$\int_0^h f(x)dx = h\left\{af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right\}$$

is exact for polynomial of as high order as possible, and determine the order of the truncation error.

53. Find the quadrature formula

$$\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$$

which is exact for polynomials of highest possible degree. Then use the formula on

$\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ and compare the exact value.

54. Find the solution of the system of equation

$$\frac{d\boldsymbol{u}}{dt} = \boldsymbol{Au}$$

where **u** = $[u_1, u_2]^T$ and **A** = $\begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix}$

55. Find the solution of the initial value problem

$$\frac{d\boldsymbol{u}}{dt} = \boldsymbol{Au}, \boldsymbol{u}(0) = [1, 0]^T$$

where **A** = $\begin{bmatrix} -2 & 1 \\ 1 & -20 \end{bmatrix}$ and $\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$. Is the system asymptotically stable?

56. Obtain a general solution of the system of equations

$$\frac{du_1}{dt} = -5u_1 + 2u_2 + t$$

$$\frac{du_2}{dt} = 2u_1 - 2u_2 + e^t.$$

57. Evaluate, I = $\int_{-\infty}^{\infty} \frac{e^{-x^2}}{x^2+x+1}$dx, by using Gauss Hermite 3-point formula.

58. Evaluate, $\int_0^{\infty} \frac{e^{-x}}{1+x^2}$dx, by using Gauss Legendre 2- point formula.

59. Evaluate, $\int_0^{\infty} \frac{e^{-x}}{1+x^2}$dx, by using Gauss Legendre 3-point formula.