RING THEORY

CORE PAPER - X

1 mark questions

1. What is a ring?
2. What is the distributive property in a ring?
3. What is a subring of a ring?
4. What is an integral domain?
5. What is a field?
6. What is the characteristic of a ring?
7. What is an ideal of a ring?
8. What is the ideal generated by a subset of a ring?
9. What are factor rings?
10. What are the operations on ideals?
11. What is a prime ideal?
12. What is a ring homomorphism?
13. What is the kernel of a ring homomorphism?
14. What is an isomorphism?
15. What is the First Isomorphism Theorem?
16. What is the Second Isomorphism Theorem?
17. What is the Third Isomorphism Theorem?
18. What is the field of quotients?
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20. What is the composition of two ring homomorphisms?
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22. What is the division algorithm for polynomials?
23. What is a principal ideal domain?
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26. What is the irreducibility test?
27. What is the unique factorization theorem in Z[x]?
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34. What is a factorial domain?
35. What is a unique factorization domain?
36. What is the Euclidean function in an integral domain?
37. What is a Euclidean domain?
38. What is a Euclidean element in an integral domain?

39. What is the principal ideal of an element in an integral domain?
40. What is the generator theorem for principal ideal domains?
41. A __________ is a set equipped with two binary operations, addition and multiplication.
42. The distributive property states that for all elements a, b, and c in a ring R, a(b+c) = ab + ac and (a+b)c = ac + bc.
43. A __________ of a ring R is a subset S of R that is closed under addition, multiplication, and additive inverses.
44. An __________ is a commutative ring with no zero divisors.
45. A __________ is a commutative ring where every nonzero element has a multiplicative inverse.
46. The __________ of a ring R is the smallest positive integer n such that n1 = 0, where 1 is the multiplicative identity of R.
47. An __________ of a ring R is a subset I of R such that for all a,b in R, a+b and ab are in I, and for all r in R, ra and ar are in I.
48. The __________ of a subset S of a ring R is the smallest ideal of R that contains S.
49. A __________ on a ring R is a binary operation that takes two ideals I and J of R and produces the ideal IJ = {ij | i in I, j in J}.
50. The quotient ring R/I consists of the equivalence classes of elements in R under the relation x ~ y if and only if x - y is in I.
51. A __________ of a ring R is a proper ideal P such that whenever ab is in P, either a or b is in P.
52. A __________ is a function f: R → S between two rings such that f(a+b) = f(a) + f(b) and f(ab) = f(a)f(b) for all a,b in R.
53. The kernel of a ring homomorphism f: R → S is the set of elements in R that map to 0 in S.
54. A __________ is a ring homomorphism that is also a bijection.
55. The First Isomorphism Theorem states that if f: R → S is a ring homomorphism, then R/ker(f) is isomorphic to the image of f in S.
56. The Second Isomorphism Theorem states that if I and J are ideals of a ring R such that I is contained in the sum I+J, then (I+J)/J is isomorphic to I/(I∩J).
57. The Third Isomorphism Theorem states that if I and J are ideals of a ring R with I contained in J, then (R/I)/(J/I) is isomorphic to R/J.
58. The field of quotients of an integral domain D is the smallest field that contains D.
59. The image of a ring homomorphism f: R → S is the subset of S consisting of all elements of the form f(r) for some r in R.
60. The composition of two ring homomorphisms f: R → S and g: S → T is the function (g ∘ f): R → T defined by (g ∘ f)(r) = g(f(r)).
61. A __________ over a commutative ring R is a polynomial in the variable x with coefficients in R.

62. The division algorithm for polynomials states that for any polynomials f and g with g nonzero, there exist unique polynomials q and r such that f = qg + r and deg(r) < deg(g).
63. A __________ is an integral domain where every ideal is generated by a single element.
64. The factorization of a polynomial f over a field K is a representation of f as a product of irreducible polynomials in K[x].
65. The __________ test states that if a polynomial f(x) = a_nx^n + ... + a_1x + a_0 has integer coefficients and p is a prime such that p does not divide a_n and p divides all the other coefficients, then f is irreducible over the integers.
66. The __________ criterion states that if a polynomial f(x) = a_nx^n + ... + a_1x + a_0 has integer coefficients and there exists a prime p such that p divides a_i for i = 0, 1, ..., n-1, p does not divide a_n, and p^2 does not divide a_0, then f is irreducible over the integers.
67. The __________ factorization theorem states that every polynomial with coefficients in Z can be factored uniquely as a product of powers of irreducible polynomials and a unit.
68. A __________ is an element of an integral domain that cannot be expressed as the product of two non-units.
69. A __________ is an element of an integral domain that is not 0 or a unit and cannot be expressed as the product of two non-units.
70. A __________ is an integral domain where every nonzero element can be factored into irreducibles in a unique way.
71. Divisibility in an integral domain D is defined by saying that a divides b (written a | b) if there exists an element c in D such that b = ac.
72. An element p in an integral domain D is __________ if whenever p divides ab, either p divides a or p divides b.
73. An element p in an integral domain D is __________ if whenever p divides ab, either p divides a or p divides b, and p is not a unit.
74. A __________ is an integral domain where every nonzero element can be factored into irreducibles.
75. A __________ domain is an integral domain where every nonzero element can be factored into primes in a unique way up to reordering and multiplication by units.
76. The Euclidean function on an integral domain D is a function f: D → N that satisfies the following properties: (i) f(a) = 0 if and only if a = 0, (ii) for any nonzero elements a and b in D, there exist elements q and r in D such that a = bq + r and either r = 0 or f(r) < f(b), and (iii) for any nonzero elements a and b in D, f(ab) = f(a)f(b).
77. A __________ domain is an integral domain where there exists a Euclidean function.
78. A __________ is an element of an integral domain D such that for any

nonzero elements a and b in D, there exist elements q and r in D such that $a = bq + r$ and either $r = 0$ or $f(r) < f(b)$.
79. The __________ of an element a in an integral domain D is the set of all elements b in D such that a divides b.
80. The __________ theorem states that every ideal in a principal ideal domain is generated by a single element.

2 marks questions

1. Define a ring and give an example.
2. State the distributive property in a ring.
3. What is a subring of a ring?
4. Define an integral domain and give an example.
5. What is a field? Give an example.
6. Define the characteristic of a ring.
7. What is an ideal of a ring?
8. Define the ideal generated by a subset of a ring.
9. What are factor rings?
10. What are the operations on ideals?
11. Define a prime ideal.
12. What is a ring homomorphism?
13. Define the kernel of a ring homomorphism.
14. What is an isomorphism?
15. State the First Isomorphism Theorem.
16. State the Second Isomorphism Theorem.
17. State the Third Isomorphism Theorem.
18. What is the field of quotients?
19. Define the image of a ring homomorphism.
20. What is the composition of two ring homomorphisms?
21. Define a polynomial ring.
22. State the division algorithm for polynomials.
23. Define a principal ideal domain.
24. What is the factorization of a polynomial over a field?
25. What is the Eisenstein criterion?
26. What is the irreducibility test?
27. State the unique factorization theorem in Z[x].
28. Define an irreducible element in an integral domain.
29. Define a prime element in an integral domain.
30. What is a unique factorization domain?
31. Define divisibility in an integral domain.
32. Define a prime element in an integral domain.
33. Define an irreducible element in an integral domain.

34. What is a factorial domain?
35. Define a unique factorization domain.
36. What is the Euclidean function in an integral domain?
37. Define a Euclidean domain.
38. Define a Euclidean element in an integral domain.
39. Define the principal ideal of an element in an integral domain.
40. State the generator theorem for principal ideal domains.

6/7 marks questions

1. Define a ring and explain the properties of rings. Give an example of a ring.
2. What is the distributive property in a ring? Prove that the distributive property holds in a ring.
3. Define a subring of a ring and give an example.
4. Explain what an integral domain is and give an example.
5. Define a field and give an example.
6. What is the characteristic of a ring? Explain with an example.
7. Define an ideal of a ring and give an example.
8. Explain the concept of the ideal generated by a subset of a ring with an example.
9. What are factor rings? Explain with an example.
10. Describe the operations on ideals in a ring.
11. Define a prime ideal in a ring and give an example.
12. What is a ring homomorphism? Explain with an example.
13. Define the kernel of a ring homomorphism and prove that it is an ideal.
14. What is an isomorphism? Give an example of an isomorphism between two rings.
15. State the First Isomorphism Theorem for rings and prove it.
16. State the Second Isomorphism Theorem for rings and prove it.
17. State the Third Isomorphism Theorem for rings and prove it.
18. What is the field of quotients? Explain with an example.
19. Define the image of a ring homomorphism and give an example.
20. What is the composition of two ring homomorphisms? Explain with an example.
21. Define a polynomial ring over a commutative ring and give an example.
22. State the division algorithm for polynomials and prove it.
23. What is a principal ideal domain? Give an example.
24. Explain the factorization of polynomials over a field with an example.
25. What is the Eisenstein criterion? Explain with an example.
26. What is the irreducibility test for polynomials? Explain with an example.
27. State the unique factorization theorem in Z[x] and prove it.
28. Define an irreducible element in an integral domain and give an example.

29. Define a prime element in an integral domain and give an example.
30. What is a unique factorization domain? Explain with an example.
31. Define divisibility in an integral domain and give an example.
32. Define a prime element in an integral domain and give an example.
33. Define an irreducible element in an integral domain and give an example.
34. What is a factorial domain? Explain with an example.
35. Define a unique factorization domain and give an example.
36. What is the Euclidean function in an integral domain? Explain with an example.
37. Define a Euclidean domain and give an example.
38. Define a Euclidean element in an integral domain and give an example.
39. Define the principal ideal of an element in an integral domain and give an example.
40. State the generator theorem for principal ideal domains and prove it.