Government (Auto.) college, Rourkela

SUBJECT-STATISTICS

PAPER- P-301

SHORT TYPE QUESTION:-

1. Cluster analysis is used for______________.
2. Define Sample mean vector?
3. Define Orthogonal factor model?
4. Define dispersion matrix?
5. Define principal component analysis?
6. Define canonical correlation and variates?
7. If X ~ Np (μ, ∑) then X +d follows_____________.
8. Let A be the matrix of constants of order m×p and given that rank(A)=m then the probability distribution of AX is___________.
9. The exponent of multivariate normal density function follows____________.
10. When referring to a multivariate random variable X, then matrix ∑ refers to the _____________ matrix
11. Let h be any fixed row vector of order n×1 and independently distributed with matrix D of order n×n then the what is the conditional distribution of h' Dh/ h' ∑h ?
12. The two vectors X1 and X2 are independent iff __________.
13. The ______________________matrix of a multivariate random variable is always symmetric.
14. The matrix S/(n-1) is a biased estimate of the variance-covariance matrix of the multivariate population. (true/false)
15. The probability distribution is said to be multivariate normal iff every linear combination of X is_____________.
16. Let $X_{p\times n}$ is random sample from N(μ, ∑). The distribution of S = Σ(X-E(X))'(X-E(X)) is_____.
17. Hotelling T^2 test is generalization of___________.
18. Define Mahalnobis D^2 statistic
19. Define Misclassification error
20. Define Fisher discriminant function.
21. A p-variate normal distribution is said to be non-singular if_____

Long-Type Questions:

1. State and prove that any two properties of Principal Component Analysis.
2. What are Principal Components? What purpose do they serve? Determine the first principal component and its variance when

   $$\Sigma = \begin{matrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{matrix}$$
3. Let X of order P×1 has mean μ and covariance matrix ∑. Obtain K-principal components of standardized vector Z of X.
4. Define canonical correlations and canonical variables. Derive the first canonical correlation coefficient and the corresponding canonical variables.
5. Let X ~ Np(μ,Σ). A random sample of size n = 5 is collected and shown below:

   $$X = \begin{bmatrix} 9 & 12 & 3 \\ 2 & 8 & 4 \\ 6 & 6 & 0 \\ 5 & 4 & 2 \\ 8 & 10 & 1 \end{bmatrix}$$. Compute:

   i) Mean vector for the data set ii) Covariance matrix of X iii) Correlation matrix of X.
6. State and prove relation between multivariate normal and χ 2 distribution.
7. Identify the sampling distribution of maximum likelihood estimators of multivariate normal distribution.Show that these maximum likelihood estimators are independently distributed.
8. Derive moment generating function and characteristic function for multivariate normal distribution.
9. Define multivariate normal distribution. Write probability density function of multivariate normal distribution
10. Let X1, X2, …. Xn be a random sample of size n from Np (μ, ∑). Find maximum likelihood estimator of μ and ∑ based on these n observations.
11. Let X ~ Np (μ, ∑) Obtain sufficient statistic for μ and ∑ where (μ, ∑) are unknown.
12. Derive the characteristic function of multivariate normal distribution from its probability density function.
13. Let X ~ Np (μ, ∑). find the distribution of (A x + b) where Ak x p matrix of constants with k ≤ p and bk x 1 vector of constant.
14. Let X ~ Np (μ, ∑) with μ' = [2, -1, 1] and ∑ = $\begin{bmatrix} 2 & 2 & 2 \\ & 4 & 3 \\ & & 3 \end{bmatrix}$
    i. Find the distribution of $[X_1+3X_2, 3X_3]'$
    ii. Find the conditional distribution of X_1+3X_2 given $3X_3=2$
    iii. Find a 2×1 vector b such that X_2 and X_2 - b' $[X_1, X_3]'$ are independent.
15. Let X_1, X_2, X_3, X_4 and X_5 be independent and identically distributed random vectors with mean vector μ and covariance matrix ∑. Find the mean vector and covariance

matrices for each of the two linear combinations of random vectors given below in terms of μ and ∑,

$\frac{1}{5}X_1 + \frac{1}{5}X_2 + \frac{1}{5}X_3 + \frac{1}{5}X_4 + \frac{1}{5}X_5$

$X_1 - X_2 + X_3 - X_4 + X_5$

Also obtain the covariance between the two linear combinations of random vectors.

16. If A ~ Wp (n, ∑) then |A| / |∑| is distributed as product of p independent chi-square variate with d.f .n, n-1,…. N – p + 1.
17. Define Hotelling T^2 .State with two examples the uses of Hotelling's T^2
18. Define Hotelling T^2 and show that it is invariant under non-singular linear transformation.

19. Let Z ~ Np (0,∑) and C has the Wishart distribution then n − p +1 X C^{-1} X ~ $F_{p,n-p+1}$ .Derive the distribution of T^2 under the null hypothesis H_0: $\mu = \mu_0$
20. Describe Characteristic function of Wishart distribution.
21. Define Wishart Matrix. State and prove any two important properties of Wishart distribution.
22. Define Wishart distribution. State and prove the additive property of canonical wishart distribution.
23. State the density function of the Wishart distribution and identify its parameter.
24. Define the Wishart distribution and describe how it generalizes the chi-square distribution
25. Derive relationship between Hotelling T2 and Mahalonobis distance.
26. show that hotelling T^2 is in variant under Non - singular transformation.
27. Derive Fisher's Discriminant function to discriminate between two populations. State the assumptions clearly.
28. Define the problem of classifying an observation into one of the two populations.
29. Define linear discriminant function and probabilities of misclassification

30. Mention any two properties of multivariate normal distribution.
31. Explain the use of partial and multiple correlation coefficients.
32. Outline the use of discriminant analysis.
33. What are canonical correlation coefficients and canonical variables?
34. Write down any four similarity measures used in cluster analysis.
35. Let $X_1, X_2, \ldots, X_n$ be independent N(0 , 1) random variables.Show that X' A X is chi-square if A is idempotent, where X= $(X_1, X_2, \ldots, X_n)'$.
36. How will you test the equality of covariance matrices of two multivariate normal distributions on the basis of independent samples drawn from two populations?

37. a) What are principal components?. Outline the procedure to extract principal components from a given covariance matrix

38. Define partial correlation between X_i and X_j .Also prove that
$r_{12.3}$= (r_{12}-$r_{13}r_{23}$)/ {√ (1-r^2_{23}) √(1-r^2_{13})}

39. Explain the method of extracting canonical correlations and their variables from a dispersion matrix.

40. Describe canonical variable and canonical correlations. State and prove any two properties of canonical variables.

41. Point out similarities and dissimilarities between principal component analysis and factor analysis.

42. Based on random sample of size n from Np (μ, ∑) .derive an expression for MLE of Σ.

43. Show that the principal components are uncorrelated and have variances equal to the eigen values of ∑.

44. Write a note on cluster analysis. Describe a method of forming clusters from given observations by using a distance function.

45. Describe minimum ECM rule. Derive the same to discriminate between and population.