GOVERNMENT AUTONOMOUS COLLEGE ROURKELA

QUESTION BANK (DESCRIPTIVE)

12-Aug-23

SUBJECT: STATISTICS **PAPER: (CORE-III) PROBABILITY AND PROBABILITY DISTRIBUTIONS**

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Short Questions (Each carry 1 mark)

1. The _____________ of a random experiment is the set of all possible outcomes.
2. The _____________ definition of probability is based on the assumption of equally likely outcomes.
3. The Law of _____________ states that the probability of the union of two events is the sum of their individual probabilities minus the probability of their intersection.
4. Conditional probability is denoted by P(A|B) and is defined as P(A and B) / P(B), where A and B are _____________ events.
5. Bayes' theorem allows us to calculate the probability of event A given event B has occurred, using the formula: P(A|B) = (P(B|A) * P(A)) / P(B).
6. A _____________ random variable can take on a countable number of distinct values.
7. The probability mass function (p.m.f.) gives the probability of each value that a _____________ random variable can take.
8. The cumulative distribution function (c.d.f.) of a continuous random variable X is defined as $P(X \leq x)$, where x is a _____________ value.
9. The joint probability mass function (p.m.f.) gives the probability of two _____________ random variables taking specific values simultaneously.
10. The _____________ of two random variables X and Y is a new random variable defined as Z = aX + bY, where a and b are constants.
11. The _____________ of a random variable X is denoted by E(X) and represents its long-term average value.
12. The _____________ generating function of a random variable X is defined as M_X(t) = E(e^(tx)).
13. The _____________ theorem for moment generating functions states that if two random variables have the same moment generating functions in an open interval around zero, they have the same distribution.
14. The _____________ function of a random variable X is defined as $\Phi X(t) = E(e^{itx})$, where i is the imaginary unit.
15. The concept of _____________ involves finding the expected value of a random variable given certain information or conditions.

16. The uniform distribution is characterized by a constant _____________ function over a specified interval.
17. The parameters of the binomial distribution are n, the number of trials, and p, the probability of _____________ in each trial.
18. The Poisson distribution is often used to model rare events with a _____________ occurrence rate.
19. The central limit theorem states that the distribution of the _____________ of a large number of independent and identically distributed random variables approaches a normal distribution.
20. The exponential distribution is commonly used to model the time between events in a _____________ process.
21. An _____________ is a subset of the sample space of a random experiment.
22. The _____________ definition of probability is based on relative frequencies observed in actual experiments.
23. The Law of _____________ states that for any two events A and B, P(A or B) = P(A) + P(B) - P(A and B).
24. Events A and B are _____________ if the occurrence of one event does not affect the probability of the other event.
25. The _____________ theorem provides a way to update probabilities when new evidence is obtained.
26. The probability density function (p.d.f.) describes the behavior of a _____________ random variable.
27. The cumulative distribution function (c.d.f.) gives the probability that a continuous random variable X takes a value _____________ or less.
28. The _____________ p.m.f. gives the probability distribution of one variable in the presence of the other variable.
29. A linear combination of random variables, Z = aX + bY, is called a _____________ of random variables.
30. The _____________ of two random variables X and Y is denoted by Cov(X, Y).
31. The _____________ of a random variable X is a measure of its center of mass.
32. The moment generating function (MGF) of a random variable X is used to find its _____________ moments.
33. The _____________ theorem states that if the MGFs of two random variables are the same in a neighborhood of zero, they have the same distribution.
34. The _____________ function of a random variable X provides a way to analyze its characteristic behavior.
35. The _____________ of a random variable Y given event A is denoted by E(Y|A).
36. In a uniform distribution, the probability density is _____________ across the interval.
37. The binomial distribution is used for counting the number of _____________ outcomes in a fixed number of trials.
38. The Poisson distribution can approximate the binomial distribution when the number of trials is large and the probability of success is _____________.
39. The central limit theorem states that the sum of a large number of _____________ random variables tends to have a normal distribution.
40. The exponential distribution is often applied to model the time between occurrences in a _____________ process. Probability density function f(x) can be used to describe the probability of a __________ random variable X**.**
41. The Cumulative distribution function of a CRV can be defined as -------------------
42. A random variable is a ________function.
43. A random variable is not a_________ function.

44. The probability mass function cannot take ________values.
45. For a DRV, the probability density function represents________.
46. The probability distribution function cannot have______ values.
47. $E(a\mathrm{X} + b)$ = ---------------------
48. $E(\mathrm{X} - \bar{X})$ = ---------------------
49. The other name of the moments about origins is ----------------.
50. The other name of the moments about mean is -----------------.
51. Binomial distribution is -------------------- if p = q = ½.
52. For, binomial distribution is variance --------- mean.
53. Binomial distribution is ---------------------
54. Geometric distribution has -----------.
55. Mean, Variance and third central moment of Poisson distribution are ________
56. Poisson distribution is not a ----------- distribution.
57. A binomial random variable is approximated to Poisson random variable when sample value is --------- and probability is close -----------.
58. Exponential distribution is a special case of -----------------.
59. For a normal distribution, coefficient of skewness is -----------------.
60. If ‘X’ is a Poisson variate such that $(X = 2) = 9(X = 4) + 90\ P(X = 6)$, the variance is ------------.
61. The graph of the normal distribution is ------------------.
62. The normal distribution is a ---------------------------- probability distribution.
63. The -------- of the normal distribution lies at the centre of normal curve.
64. The MGF of Binomial distribution is -------------.
65. $(X\mathrm{Y}) = EE(X)E(Y)$ if X and Y are ------------ variables.

2 Marks Questions

1. Define the sample space and give an example of a random experiment along with its sample space.
2. Explain the difference between a classical, statistical, and axiomatic definition of probability.
3. State and prove the Law of Addition for Probability.
4. What are independent events? Provide an example to illustrate.
5. State Bayes' theorem and explain its significance in conditional probability calculations.
6. Define a random variable. Differentiate between discrete and continuous random variables with examples.
7. Explain the terms: probability mass function (p.m.f.), probability density function (p.d.f.), and cumulative distribution function (c.d.f.).
8. Given the joint p.m.f. of a bivariate random variable, how do you find the marginal p.m.f.?
9. Discuss the concept of transformation of random variables. Provide an example.
10. What is meant by the independence of random variables? How can you determine if two random variables are independent?
11. Define the mathematical expectation of a random variable. State and prove its linearity property.
12. What are the moment generating function and the characteristic function of a random variable? How are they useful?
13. Explain the concept of conditional expectation and its applications.
14. State the uniqueness theorem for moment generating functions.
15. If X and Y are bivariate random variables, how is the covariance between them defined? State its properties. Describe the characteristics and use cases of the uniform distribution.
16. Provide an example scenario where the binomial distribution is applicable. State its parameters.
17. Explain the limiting cases of the Poisson distribution. When is it commonly used?
18. Define the normal distribution. What is the significance of the central limit theorem?
19. Discuss the properties and applications of the exponential distribution.
20. If a fair six-sided die is rolled, what is the sample space for this random experiment? List all the possible outcomes.
21. Differentiate between mutually exclusive and exhaustive events. Provide an example.
22. How is conditional probability defined? Give an example to illustrate.
23. State the Law of Multiplication for Probability. Provide a scenario where it can be applied.
24. Explain the concept of complementary events. How are they related to the probability of an event?
25. Give an example of a random variable that follows a discrete probability distribution. State its p.m.f.
26. Define the concept of joint probability distribution for two random variables.
27. If X is a continuous random variable, how is the cumulative distribution function (c.d.f.) of X defined?

28. Discuss the concept of independence in the context of two-dimensional random variables.
29. How can the distribution of a transformed random variable be obtained? Provide a transformation example.
30. Calculate the mathematical expectation of a discrete random variable X given its p.m.f.

31. What is the relationship between moments and cumulants of a random variable?
32. Explain the concept of moment generating function. How does it help in finding moments?
33. State the inversion theorem for moment generating functions.
34. In what situations is the characteristic function of a random variable useful?
35. Discuss the properties of the binomial distribution. Provide an example scenario where it is applicable.
36. Define the parameters of the Poisson distribution. How does it approximate the binomial distribution?
37. Explain the concept of the central limit theorem. How does it relate to the normal distribution?
38. Calculate the mean and variance of an exponential distribution with a given rate parameter.
39. How can the gamma distribution be useful in modeling waiting times or survival times?
40. In a deck of 52 playing cards, what is the probability of drawing a heart or a spade?
41. A bag contains 5 red balls and 3 green balls. If two balls are drawn without replacement, what is the probability that both balls are red?
42. If the probability of rain on any given day in a certain city is 0.3, what is the probability that it won't rain on that day?
43. A fair six-sided die is rolled. Find the conditional probability of rolling an odd number given that the result is greater than 2.
44. In a class, 60% of students play football and 40% play cricket. If 25% play both sports, what percentage of students play at least one of the two sports?
45. A discrete random variable X follows a binomial distribution with n = 10 trials and success probability p = 0.2. Find P(X = 3).
46. The continuous random variable X follows an exponential distribution with a mean of 5. Calculate the probability that X is less than 3.
47. Two dice are rolled. Let X be the sum of the two numbers rolled. Find the probability distribution of X.
48. The joint probability distribution of two random variables X and Y is given by:

49. X\Y	**50. 1**	**51. 2**
52. 0	53. 0.1	54. 0.2
55. 1	56. 0.3	57. 0.2

58. Calculate the marginal probability distribution of Y.
59. If the random variable Z is defined as Z = 2X - Y, where X and Y are independent random variables with means 3 and 2, respectively, find E(Z).

60. Calculate the expected value of a random variable X with the following probability distribution:

61. X	**62. 1**	**63. 2**	**64. 3**
65. P(X)	66. 0.2	67. 0.4	68. 0.4

69. Find the moment generating function of a random variable X with the probability distribution given above.
70. If the moment generating function of a random variable X is M X(t) = $e^{2t} + 3e^{t}$, find E(X) and $E(X^2)$.
71. The random variable X follows a normal distribution with mean 10 and variance 25. Find the probability that X is between 5 and 15.
72. Calculate the cumulative distribution function (c.d.f.) of a random variable Y that follows an exponential distribution with a rate parameter of 0.1.
73. If X and Y are random variables having the joint density function f(x,y) = 1 8 (6-x-y),0 < x < 2, 2 < y < 4. Find P(X + Y < 3)
74. Calculate the expectation of a random variable X with probability density function f(x) = 2x for $0 \leq x \leq 1$.
75. Question 2: Given two random variables X and Y with joint probability distribution function f(x, y) = 3x^2y for $0 \leq x \leq 1$ and $0 \leq y \leq 1$, calculate the covariance between X and Y.
76. Define the mathematical expectation of a random variable.
77. What is the expected value of a constant?
78. Explain the linearity of expectation.
79. Calculate the expectation of a Bernoulli random variable.
80. What is the expectation of the sum of two independent random variables?
81. Define conditional expectation.
82. What is a moment generating function (MGF)?
83. How can you find the MGF of a sum of independent random variables?
84. What is a cumulant generating function (CGF)?
85. How is the CGF related to the MGF?
86. Define the nth moment of a random variable.
87. Calculate the first three moments of a Poisson random variable.
88. What are cumulants, and why are they useful in probability theory?
89. How are cumulants related to moments?
90. State the uniqueness theorem for moment generating functions.
91. Explain the inversion theorem for moment generating functions.
92. How can you use the inversion theorem to find the distribution of a random variable?
93. State the uniqueness theorem for characteristic functions.
94. Provide an example of using moment generating function in solving a real-world problem.
95. How can moment generating functions be useful in hypothesis testing?
96. Explain the concept of conditional expectation with respect to a sigma-algebra.
97. What is the law of iterated expectations, and how is it used in probability theory?

98. Calculate the conditional expectation of a random variable given another random variable.
99. Define the concept of a conditional probability density function (PDF).
100. State the properties of expectation for constants and random variables.
101. What is the relationship between covariance and the expectation of the product of two random variables?
102. Explain the concept of the law of total expectation.
103. Calculate the MGF of a standard normal random variable.
104. How do you compute higher moments from the MGF?
105. Define the CGF, and explain its relationship to the MGF.
106. What is the characteristic function of a random variable?
107. How does the characteristic function relate to the probability density function (PDF) of a random variable?
108. Calculate the characteristic function of a uniform random variable.
109. Provide an example of using characteristic functions in solving a practical problem.
110. How can the properties of expectation and generating functions be applied in finance or risk assessment?
111. Problem 1: Expectation of a Random Variable Let X be a random variable with the probability density function: $f(x) = 2x$ for $0 \leq x \leq 1$. Calculate E(X), the expectation of X.
112. Problem 2: Linearity of Expectation Suppose you have two independent random variables, X and Y, with $E(X) = 3$ and $E(Y) = 5$. Calculate $E(2X + 3Y)$.
113. Problem 3: Moment Generating Function (MGF) Find the MGF of a standard normal random variable, Z, i.e., MZ(t).
114. Problem 4: Conditional Expectation Let X and Y be two random variables with the following joint probability distribution function: $f(x, y) = 6xy$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Calculate $E(X \mid Y = 1/2)$.
115. Problem 5: Characteristic Function Calculate the characteristic function $\phi(t)$ of a random variable with the probability density function: $f(x) = (1/\pi) * (1 + x^2)^(-1)$ for $-\infty < x < \infty$.
116. Problem 6: Cumulant Generating Function (CGF) Given the MGF of a random variable X as $M(t) = e^(2t + 3t^2)$, find the CGF of X.
117. Problem 7: Uniqueness Theorem for Characteristic Functions Show that if two random variables have the same characteristic function, their probability distributions are the same.
118. Problem 8: Law of Total Expectation Suppose you have three mutually exclusive events A, B, and C with probabilities $P(A) = 0.3$, $P(B) = 0.4$, and $P(C) = 0.3$. You have a random variable X that is defined as follows:

a) $X = 2$ if event A occurs.

b) $X = 3$ if event B occurs.

c) $X = 4$ if event C occurs.

119. Calculate E(X), the expectation of X.
120. Problem 9: Conditional Expectation and Variance Let X be a random variable with $E(X) = 5$ and $Var(X) = 4$. Find $E(X \mid X > 7)$, the conditional expectation of X given that X is greater than 7.
121. Define the probability mass function (PMF) of a discrete random variable.
122. Calculate the mean (expected value) of a binomial random variable.

123. What is the Poisson distribution, and when is it commonly used?
124. Explain the geometric distribution and its parameters.
125. Find the variance of a binomial random variable.
126. Describe the properties of a uniform discrete random variable.
127. What is the limiting case of the binomial distribution when the number of trials becomes large?
128. Define the probability density function (PDF) of a continuous random variable.
129. Calculate the median of a normal distribution.
130. Explain the exponential distribution and its applications.
131. What are the parameters of a beta distribution?
132. Find the standard deviation of a normal distribution.
133. Describe the properties of a uniform continuous random variable.
134. What is the limiting case of the normal distribution when the sample size becomes large?
135. Define the gamma distribution and its shape parameter.
136. Explain the Central Limit Theorem and its significance in statistics.
137. Calculate the area under the standard normal distribution curve (z-score) between $z = -1.5$ and $z = 2.0$.
138. In an exponential distribution with a rate parameter $\lambda = 0.1$, find the probability that the random variable X is greater than 5.
139. If a random variable Y follows a beta distribution with parameters $\alpha = 2$ and $\beta = 3$, compute $P(Y < 0.6)$.
140. Find the median of a uniform continuous random variable X that ranges from 0 to 10.
141. When can you use the Poisson distribution to approximate a binomial distribution?
142. Describe the connection between the exponential distribution and the Poisson process.
143. What is the continuity correction in approximating discrete distributions with continuous ones?
144. In a binomial distribution with $n = 8$ trials and $p = 0.3$, calculate the probability of getting exactly 3 successes.
145. A Poisson process has an average rate of 4 events per hour. What is the probability that exactly 2 events occur in the next 30 minutes?
146. In a geometric distribution with a success probability of 0.2, find the probability that the first success occurs on the fifth trial.
147. Suppose you have a uniform discrete random variable X that can take values between 1 and 6 inclusive. What is the mean (expected value) of X?
148. How does the normal distribution approximate the binomial distribution under certain conditions?

149. Problem 10: Moment Calculation Calculate the third moment (central moment) of a random variable Y with the following probability mass function: $P(Y = -1) = 0.2$ $P(Y = 0) = 0.5$ $P(Y = 1) = 0.3$

Long Questions (Each carry 7 Marks)

1. Explain the concepts of sample space, events, and algebra of events in the context of probability. Provide examples to illustrate each concept.

2. Discuss the differences between classical, statistical, and axiomatic definitions of probability. Highlight the situations where each definition is most applicable.

3. Define conditional probability and demonstrate its calculation using an example. How does conditional probability help in solving real-world problems involving uncertain events?

4. State and prove the laws of addition and multiplication of probabilities. Provide intuitive explanations and examples to support your proofs.

5. Explain the concept of independent events. Show how to determine whether two events are independent or not, and discuss its significance in probability calculations.

6. Describe the theorem of total probability and its importance in solving problems involving multiple scenarios. Provide a practical example to showcase its application.

7. Derive Bayes' theorem from the definitions of conditional probability and the multiplication rule. Provide a step-by-step explanation and show how it can be used to update probabilities based on new information.

8. If A and B are any two events (subsets of sample space S) and are not disjoint, then Prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

9. For n events A_1, A_2,……,An, we have

$$P(\cup_{i=1}^{n} Ai) = \sum_{i=1}^{n} P(Ai) - \sum\sum_{1 \le i < j \le n} P(Ai \cap Aj) + \sum\sum\sum_{1 \le i < j < k \le n} P(A_i \cap A_j \cap A_k)$$

$$+\ldots+(-1)^{n-1} P(A_1 \cap A_2 \ . \ . \ . \cap A_n)$$

10. A random variable X has the probability function $(x) = 1/2\text{^}x$, $x = 1,2,3, \ldots$Find the MGF, mean and variance.
11. A continuous random variable X has a pdf $(x) = 3x^2, 0 \le \ \le 1.$
    Find a and b such that
    a. $P(X \le a) = P(X > a)$ *and* $b. P(X > b) = 0.05.$
12. For the distribution defined by the pdf

$$f(x) = \begin{cases} x, 0 < x < 1 \\ 2 - x, 1 < x < 2 \\ 0, otherwise \end{cases}$$

Compute the r th moment about the origin. Hence deduce the first four moments about mean.

13. Let X be a random variable with pdf $f(x) = \begin{cases} \frac{1}{3} e^{\frac{-x}{3}}, x > 0 \\ 0, otherwise \end{cases}$

    Find a) P (X > 3) b) MGF of X c) E(X) d) Variance(X).

14. Six dice are thrown 729 times. How many times do you expect at least 3 dice to show a 5 or 6?
15. Find MGF of binomial distribution. Hence derive mean, variance and standard deviation.

16. A manufacturer of cotton pins knows that 5% of his product is defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective. What is the approximate probability that a box will fail to meet the guaranteed quality?
17. Fit a poisson distribution to the following data, which gives the number of yeast cells per square for 400squares.

No. of cells per square (x)	0	1	2	3	4	5	6	7	8
No. of squares (f)	103	143	98	42	8	4	2	0	0

18. State and prove the memory less property of Geometric distribution.
19. An item is produced in large numbers. The machine is known to produce 5% defectives. A quality control inspector is examining the items by taking them at random. What is the probability that at least 4 items are to be examined in order to get 2 defectives?
20. Find MGF, mean and variance of Uniform distribution.
21. State and prove the memory less property of Exponential distribution.

22. Suppose that the life of an industrial lamp, in thousands of hours, is exponentially distributed with

    Failure rate λ = 1/3 Find the probability that the lamp will last

    a. Longer than its mean life of 3000 hours.
    b. Between 2000 and 3000 hours
    c. For another 1000 hours given that it is operating after 2500 hours.

23. In a certain city, the daily consumption of electric power in millions of kilowatt hours can be treated as a gamma random variable with parameters $\nu = 3$ $annd$ λ = 1/2 . If the power plant of this city has a daily capacity of 12 million kilo-watt hours, what is the probability that this power supply is inadequate on any given day?
24. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and Standard deviation of the distribution.
25. 17. In an examination, it is laid down that a student passes if he secures 30% or more marks. He is placed in first, second and third division according as he secures 60% or more marks, between 45% and 60% marks and marks between 30% and 45% respectively. If he secures 80% or more marks, he gets distinction. It is noticed from the results that 10% of the students failed and 5% of them obtained distinction. Assuming normal distribution of marks, what percentage of students placed in the second division?
26. A random variable X has a uniform distribution over the interval (-3,3).

    Find a. P[X = 2]

    b. P[X < 2]

    c. P [|X| < 2]

    d. P [|X – 2| < 2] and

    e. Find K such that P [X > K] = 1/3.

27. If X is a normal variable with mean 30 and S.D = 5, then

    Find a. P [26 < X < 40]   b. P [X > 45]   c. P [|X – 30|>5]

28. Let X and Y be two random variables having the joint probability function f(x ,y) = k(x+2y) where X and Y can assume only integer values 0,1 and 2. Find the marginal and conditional distributions.

29. From the following bivariate probability distribution,

Y/X	-1	0	1
0	1 / 15	2 / 15	1 / 15
1	3 / 15	2 / 15	1 / 15
2	2 / 15	1 / 15	2 /1 5

Find a). Marginal distributions of X and Y b). Conditional distributions

30. If $ff(x) = e^{-(x+y)}$, $0 \le x, y \le \infty$ is the joint pdf of random variables X and Y, find a. P [X < 1] b. P [X > Y
c. P [X + Y $\le$ 1]

31. Two continuous random variables X and Y have the joint p. d. f

$$F(x, y) = \begin{cases} 6\,(1- x- y),\ x > 0,\ y > 0\ ,\ 0 < x+y < 1 \\ 0, \text{ otherwise} \end{cases}$$

Find the marginal distributions and conditional distributions of X and Y. Hence examine whether X and Y are independent.

32. If X and Y are independent random variables each normally distributed with mean zero and variance
σ^2, find the density function of R = $\sqrt{x^2 + y^2}$ and $\theta =$ tan^(−1)(x/y)

33. Find Correlation co-efficient between X and Y from the following data:

X	78	89	97	69	59	79	61	61
Y	125	37	156	112	107	136	123	108

34. If X and y are uncorrelated random variables with variances 16 and 9, find the correlationcoefficient between x + y and x – y.
35. Find the two lines of regression for the following data.

X	150	152	155	157	160	161	164	166
Y	154	156	158	159	160	162	161	164

36. The two lines of regression are $8x - 10y = -66$, $40x - 18y = 214$. The variance of X is 9. Find the mean values of X and the correlation co-efficient between X and Y.
37. The life time of a certain brand of a tube light may be considered as a random variable with 1200 hours and SD 250 hours. Find the probability using CLT, that the average life time of 60 lights exceeds 1250 hours.
38. A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using CLT, with what probability can we assert that the mean of the sample will not differ from μ = 60 by more than 4.
39. A distribution with unknown mean μ has variance equal to 1.5. Use CLT , to determine how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be with in 0.5 of the population mean.

40. Find the Mean and Variance of the Binomial distribution
41. 2 (a)Ten coins are thrown simultaneously. Find the probability of getting at least seven heads
42. (b) The mean and variance of a binomial variable X with parameters n and p are 16 and 8. Find P X(≥1) and P X(> 2)

43. 3 (a) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 success are 0.4096 and 0.2048. Find the parameter 'p' of the distribution.
44. (b) Out of 800 families with 5 children each, how many would you expect to have
45. (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys? Assume equal probabilities for boys and girls.
46. 4) Find the Mean and Variance of the Poisson Distribution 12
47. 5 (a) 2 % of the items of a factory are defective. The items are packed in boxes. What is the probability that there will be (i) 2 defective items (ii) at least three defective items in a box of 100 items.
48. (b) A hospital switch board receives an average of 4 emergency calls in a 10 minute interval. What is the probability that (i) there are at most 2 emergency calls in a 10 minute interval (ii) there are exactly 3 emergency calls in a 10 minute interval.
49. 6 (a) If a Poisson distribution is such that P(X=1) $\frac{3}{2}$ = P(X=3) then find (i) P X(≥ 1)
50. (ii) P X(≤ 3) (iii) P X ($2\ 5 \leq \leq$) .

(b) If x is a Poisson variate such that 3p (X = 4) = $\frac{1}{2}$ P(X =2) + P(X =0)

find (i) the mean of x (ii) P X(≤ 2)

51. Find the Mean and Variance of a Normal Distribution.
52. (a) What are the main Characteristic equations of the Normal Distribution
    (b) For a normally distributed variate with mean 1 and standard deviation 3, find the probabilities that
    (i) $3.43 \leq X \leq 6.19$
    (ii) $-1.43 \leq X \leq 6.19$
53. In a Normal distribution, 7% of the items are under 35 and 89% are under 63. Find the mean and standard deviation of the distribution. 6 M (b) If X is a normal variate, find the area A (i) to the left of z = -1.78 (ii) to the right of z = -1.45 (iii) corresponding to $-0.8 \leq Z \leq 1.53$
54. The marks obtained in Mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%.Determine
55. How many students got marks above 90%
56. What was the highest mark obtained by the lowest 10% of the students
57. Within what limits did the middle of 90% of the students lie.
58. Given that the mean heights of the students in a class is 158 cms with standard deviation of 20 cm s. Find how many students heights lie between 150 cms and 170 cms, if there are 100 students in the class
59. A recent college graduate is moving to Houston, Texas to take a new Job, and is looking to purchase a home. Since Greater Houston takes in a relatively large metropolitan area of nearly 7,000,000 people, there are many homes from which to choose. When consulting the real estate web sites, it is possible to select the price range of housing in which one is most interested. Suppose the potential buyer species $200,000 to $250,000, and the result returns 105 homes with prices distributed uniformly throughout that range. Please answer the following questions

    a. What would be the probability density function that best describes this distribution of housing prices?

b. What are E(x) and σ?

c. If the buyer ultimately selects a home randomly from this initial list of 105 homes, what is the probability she will have to pay more than $235,000?

60. Referring to the above question, what is the minimum price the buyer would pay if she refines her focus to the upper 25% of the price range from $200,000 to $250,000? Note that it is possible to omit the min= and max= notation as long as the arguments are placed in the same order; that is, as long as the lower bound is placed first, the upper bound is placed immediately after it.
61. Let x be a normally-distributed random variable with a mean $\mu = 25$ and standard deviation $\sigma = 5$, answer the following questions.

a. What is the probability that x will be less than or equal to 35?

b. What is the probability that x will be less than or equal to 32?
c. What is the probability that x will be less than or equal to 30?
(d) What is the probability that x will be less than or equal to 25?

62. If x is a normally-distributed random variable with a mean $\mu = 63$ and standard deviation $\sigma = 4.5$, answer the following questions.

a. What is the probability that x will be less than or equal to -57?

b. What is the probability that x will be less than or equal to -60?

c. What is the probability that x will be less than or equal to -63?

d. What is the probability that x will be less than or equal to -70?

63. If x is a normally-distributed random variable with a mean $\mu = 36$ and standard deviation $\sigma = 3$, answer the following questions.

a. What is the probability that x will be greater than 42?

b. What is the probability that x will be greater than 39?

c. What is the probability that x will be greater than 33?

d. What is the probability that x will be greater than 27?

64. Let x be a normally-distributed random variable with a mean of $\mu = 100$ and a standard deviation of $\sigma = 15$.

a. If the area to the left of x is 0.99, what is x?

b. If the area to the left of x is 0.975, what is x?

c. If the area to the left of x is 0.95, what is x?

d. If the area to the left of x is 0.90, what is x?

65. Let x be a normally-distributed random variable with a mean of $\mu = -280$ and a standard deviation of $\sigma = 35$.

   a. If the area to the right of x is 0.10, what is x?

   b. If the area to the right of x is 0.05, what is x?

   c. If the area to the right of x is 0.025, what is x?

   d. If the area to the right of x is 0.01, what is x?

66. According to British weather forecasters, the average monthly rainfall in London during the month of June is $\mu = 2.09$ inches. Assume the monthly precipitation is a normally-distributed random variable with a standard deviation of $\sigma = 0.48$ inches.

   a. What is the probability that London will have between 1.5 and 2.5 inches of precipitation next June?

   b. What is the probability that London will have 1 inch or less of precipitation?

   c. If London authorities prepare for flood conditions when the monthly precipitation falls in the upper 5% of the normal June amounts, how much rain would have to fall to cause local authorities to begin flood preparations?

67. The time required by students enrolled in a pre-medical program to complete an organic chemistry final examination is normally-distributed with a mean of $\mu = 200$ minutes and standard deviation of $\sigma = 20$ minutes.

   a. What is the probability a student will complete the examination in 180 minutes or less?

   b. What is the probability a student will take between 180 and 220 minutes to complete the examination?

   c. Since this particular class is a large lecture section of 300 students, and the final examination period lasts 240 minutes, how many students would we expect to submit the completed exam on time?

68. large commerical agricultural concern in Spain produces melons with a diameter that is normally-distributed with a mean of $\mu = 15$ centimeters (cm) and a standard deviation of $\sigma = 2$ cm.

   a. What is the probability that a melon will have a diameter of at least 12 cm?

   b. What is the probability that a randomly selected melon will have a diameter of no less than 12 cm but no more than 16 cm?

c. The producer has an arrangement with one retailer by which they receive a slightly higher price for melons with a diameter falling in the top 10%. What is the minimum diameter a melon must have in order to qualify for the higher price?

69. A variable is exponentially-distributed with a mean of $\mu = 5$.

a. What is the form of the probability density function?

b. What is the cumulative probability function?

c. What is $p(x \leq 2)$?

d. What is $p(x \leq 4)$?

e. What is $p(x \leq 5)$?

f. What is $p(x \leq 8)$?

g. What is $p(x \leq 15)$?

70. Referring to the preceding exercise, answer the following questions.

a. What is $p(x \geq 2)$?

b. What is $p(x \geq 4)$?

c. What is $p(x \geq 5)$?

d. What is $p(x \geq 8)$?

e. What is $p(x \geq 15)$?

71. Referring to the previous exercise, answer the following questions.

a. What is $p(2 \leq x \leq 4)$?

b. What is $p(2 \leq x \leq 5)$?

c. What is $p(2 \leq x \leq 8)$?

d. What is $p(2 \leq x \leq 15)$

72. The number of visits to the Book4Less.com discount travel website is a Poisson-distributed random variable with a mean arrival rate of 10 visits per minute.

a. If the Poisson arrival rate is 10 visitors per minute, what is the mean of the associated exponential probability density function?

b. What is the exponential probability density function?

c. What is the cumulative probability function?

d. What is the standard deviation of the distribution?

73. Referring to the preceding exercise, answer the following questions.

    a. If the internet server experiences a brief power failure of 18 seconds duration during which time people would be denied access to the website, what is the probability that no one attempted to visit the Book4Less.com website anyway and thus no business was lost during the down-time? Use the exponential framework to answer this question.

    b. Answer question (a) using the Poisson probability approach. Confirm that the answer is equal to that in (a).

    c. Comment on the fact that the answers to parts (a) and (b) are exactly the same regardless of the approach (Poisson or exponential) employe